

Calculus Teaching Based on Steam Education Philosophy Taking the Teaching of Limit Concept as an Example

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ABSTRACT: STEAM education philosophy is based on mathematics. It is particularly important to promote and develop STEAM education philosophy through the wide application of mathematics. Calculus is an important course offered by university. With the implementation of popular education, the level of college students is uneven and the problems in calculus teaching are increasingly prominent. Taking the teaching of limit concept as an example, many college students can carry out limit operation, but they do not understand the philosophy contained in the limit concept and they can't innovate the application of limit. Therefore, it is the key to teaching reform to find the cognitive difficulties of students in the concept of limit. Using Geogebra as teaching tool, the mathematics culture and mathematics history are integrated into the teaching design, so that students can experience the fun of mathematics course and feel the application value of interdisciplinary. In this study, questionnaire survey, interview and experimental comparison are used to study the teaching practice by 2 teachers and 80 freshmen. The result shows that teaching content integrating into the history of mathematics and mathematics culture, teaching with Geogebra improves the students' enthusiasm, their learning achievement and promotes the professional development of teachers. The following conclusions are drawn: 1. The understanding of limit concept is one-sided, some even wrong. 2. Based on STEAM education philosophy, new teaching design can improve students' 4 learning stages based on APOS theory and then cultivate students' habit of lifelong learning. 3. The teaching design based on STEAM education philosophy can promote the professional development of teachers.

KEYWORDS: STEAM Education Philosophy, Limit Concept, Teaching Case, Teacher Professional Development

INTRODUCTION

Higher mathematics course is a compulsory public course for undergraduate students of science and engineering. The teaching goal is to enable students to master the basic theories and methods of mathematics, especially the way of thinking, master knowledge and skills and develop intelligence as well as creative ability (Epstein 2013). Calculus knowledge is the key knowledge in higher mathematics. Limit is the theoretical basis of calculus, which is closely related to continuity, derivative, differential and integral. The learning of the concept of limit includes the idea of infinite approximation, the idea of convergence and strict limit language. Limit is a basic tool to study the changing trend of variables, and a basic method for people to understand infinity from the limited (Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. 2010). In essence, it is a reflection of the qualitative change caused by the quantitative change of the objective world. However, due to the abstraction of the concept of limit, students have some difficulties in their first contact and learning. Previous studies have shown that college students have inherent and common cognitive difficulties or obstacles in learning the concept of limit, especially in the cognition of limit language (JiangTao 2018). Therefore, it is difficult for students to master the basic concept, theory and method of limit. The existing teaching methods mainly take textbooks as the carrier to teach concepts and transfer knowledge (Davis and Vinner 1986; Williams 1991). But for students to understand and grasp the concept of limit, the promotion of learning interest and the ability to analyze and solve problems, there are still some deficiencies. Therefore, to understand the current situation of college students' cognition of the concept of limit, the problems existing in the understanding of the concept of limit and how to carry out the teaching reform of the concept of limit are the problems that need to be paid attention to in current teaching.

Some researchers have already put forward that there are some difficulties in the teaching of the concept of limit. He Wenge (2005) believes that the main reason why the concept of limit is difficult to teach is that "ε-N" in definition, the abstractness of symbolic terms such as \exists , \forall , ε , N and the logical relationship or dependency between symbols in the definition. According to the author's years of teaching experience, the class hours of higher mathematics is limited, but the amount of knowledge is large. If there is no preparation before explaining the definition of limit that is, students have no abstract understanding of the size of the distance in their minds, teachers will feel like students don't understand it. Students are used to the habit of doing exercises and

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thinking methods in middle school, they are not adapted to the learning methods in university, and they have no interest and enthusiasm in learning mathematics (Collins 2004).

Some researchers have already put forward the improvement methods and strategies of limit concept teaching. For example, with the help of intuitive examples to help students understand the abstract mathematical symbols and terms in the definition of limit, explain the meaning of symbols and the logical relationship between symbols in the definition of limit (Gadanidis 2013). The group teaching method is adopted. The students in the group discuss and solve problems together in class. After class, they finish their homework together in groups every week. The students work according to their roles, cooperate with each other, discuss with each other and write reports (Zhang Jingzhong 2014). With the help of historical stories, teachers can guide students to understand the concept of limit, that is, to integrate the learning of mathematical concepts and theories into its historical development process by using historical generation method, so that students can have a resonance in the process of learning, which is convenient for students to understand and accept new knowledge (Ofir, R 1991). The concept based teaching method not only ensures students' problem-solving skills, but also deepens students' understanding. Students can realize the relevance between meticulous formulas and basic knowledge. At the same time, students need to show and explain the different methods they use in solving problems. Homework includes some questions that require students to explain answers and use concepts. The correct answer is valuable only when the students can give a reasonable explanation (Artigue M 1991).

This study explores the application of information technology and mathematical culture in STEAM education mode, which not only helps to deeply explore the connotation of STEAM education concept in mathematical culture, fully show the interdisciplinary nature of mathematical culture, but also can establish the connection between mathematics and science, technology, engineering, art and other disciplines, and promote the organic integration of STEAM education concept and mathematical education. Taking the concept of limit as the research object, based on APOS theory, this paper decomposes the psychological construction of the concept into four stages: action, process, object and schema. By means of questionnaire and interview, integrates mathematical culture and history into the teaching design, classroom teaching designs corresponding computer activities for students, such as the formal definition of limit. Finally, the decomposition of the limit concept is modified according to the teaching experiment results, and it is pointed out that the key to understand the limit concept is to coordinate the two infinite approaching processes in the limit concept.

The purpose of this paper is to understand the students' difficulties in learning the concept of limit and their misunderstanding of the concept of limit. Combined with STEAM education concept, further design limit concept teaching to avoid students' difficulties and misunderstanding. Try to find the combination of educational theory development and educational practice innovation. This paper designs a limit concept questionnaire and experiment based on APOS theory to investigate the students' cognitive level, to find out the reasons that affect the concept cognition and to explore the teaching strategies to improve the concept teaching.

THEORETICAL FRAMEWORK

In the late 1980s, American mathematician Dubinsky put forward APOS theory which can reveal students' cognitive process of abstract mathematical concepts. Dubinsky also studied the difference between process and object, and expanded it into four psychological processes: action, process, object and schema (Jim & Cottrill 1996). APOS theory originates from Piaget's study of reflective abstract mental mechanism and its promotion in the concept of higher mathematics. Dubinsky believes that mathematical knowledge is acquired by individuals in the process of solving perceived mathematical problems. In this process, individuals establish psychological activities, procedures and objects in order, and finally organize them into a schema structure to understand the problem situation (Weber K 2002). The activity stage is a necessary condition for students to understand concepts. Through the activities, students can experience the relationship between background and concepts. In the process stage, students think about the activities and experience the process of internalization and compression of thinking. They describe and reflect on the activities in their minds and abstract the unique properties of concepts. In the object stage, they realize the essence of concepts through the previous abstraction and give them formal definitions and symbols to make them refined and become a specific object. In the future study, take this as the object to carry out new activities. The formation of schema stage needs to be further improved through long-term learning activities. The initial schema includes special cases, abstract processes, definitions and symbols of reaction concepts. After learning, it establishes connections with other concepts, rules, graphics and forms a comprehensive psychological schema in mind (Asialia 1996, Dubinsky & McDonald 2001).

The concept of process is related to the theory of APOS, and Tall (2006) drew a diagram to illustrate it:

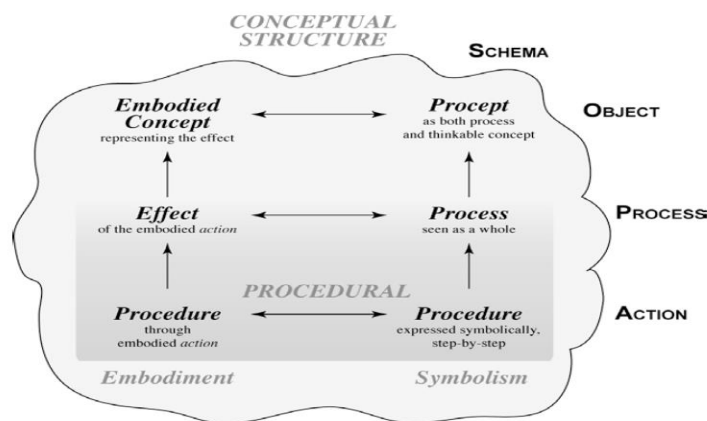


Figure 1. Procedural knowledge as part of conceptual knowledge (Tall 2006)

APOS theory not only points out that students' learning process is construction, but also indicates the level of construction. Generally speaking, these four steps can't be overstepped, because they should be advanced step by step. At the same time, we must not only stay in the concrete, intuitive and visual stage, but also sublimate, abstract and formalize, and finally complete the establishment of mathematical concepts.

STEM education originated from the National Science Foundation of the United States in 1986(CuiHong 2017). On the program of integration of science, mathematics, engineering and technology education, it was proposed. Georgette Yakman creatively integrates art, the educational idea of "STEM + Arts = STEAM" is gradually rising and developing (Mustafa N 2016). STEAM education concept can be understood as "based on mathematical elements, using science and technology to interpret art and Engineering (Zhao & Lu 2016)". Mathematics as the basic disciplines of STEAM education include science, technology, engineering and arts. It can integrate the other four subjects. The integration of disciplines and interdisciplinary integration are realized. That is, STEAM. It is an important breakthrough for the concept of education to enter mathematics education (Lyn D. 2015).

STRATEGY OF TEACHING THE CONCEPT OF LIMIT

1. The design of limit concept teaching based on STEAM

1.1 Introduce the history and importance of the concept of limit

The paper shows the importance of limit in calculus to students in PPT. Limit is the main basis for discussing function differential and integral. The limit theory is completely established through the work of Polchano, Cauchy, Vilstras in the 19th century. These people's representative work marks the establishment of limit theory. But the emergence of limit thought should be traced back to that proposed by Zhuangzi in 400 BC more than 2000 years ago (Huang Xinyu 2020).

1.2 Show the limit thought through GeoGebra

An ordered sequence of numbers $a_1, a_2, a_3, \dots, a_n$ is called a series. It is a special kind of function $f(n) = a_n$ whose domain of definition is a positive integer set.

Circle cutting is an application of limit thought. "If you cut it carefully, you will lose less. If you cut it again, you will not be able to cut it. If you combine it with the circle, you will lose nothing. The basic idea is to replace the area of the circle with the area of the inscribed regular polygon of the circle with radius. When the number of sides of the inscribed regular polygon increases, the area of the polygon is approximately equal to the area of the circle. (Figure 2 and 3)

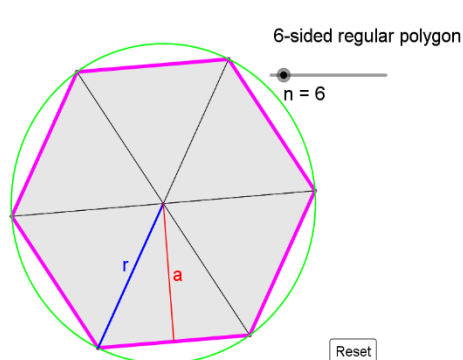


Figure 2. 6-sided regular polygon

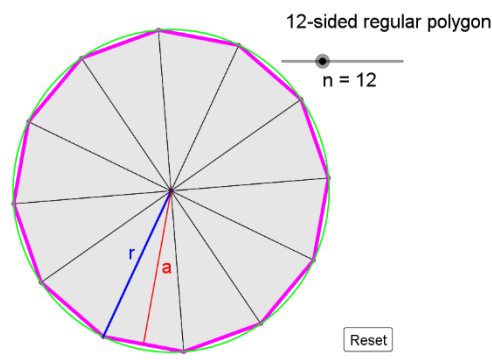


Figure 3. 12-sided regular polygon

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At present, there are infinite numbers in the series we have contacted. So with the increasing subscripts of the series, what is the trend of the value change of the series?

In “*Zhuangzi, the world*”, Zhuangzhou in the Warring States period of China mentioned "one foot hammer, half of it every day, and everything is inexhaustible". If you take half of it every day, you'll never be able to take it with a stick (Guo yufeng & Shi Ningzhong, 2012).

$$1, \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \dots, \frac{1}{2^n}$$

For the corresponding series above, let's observe the change trend as shown in the figure 4 below.

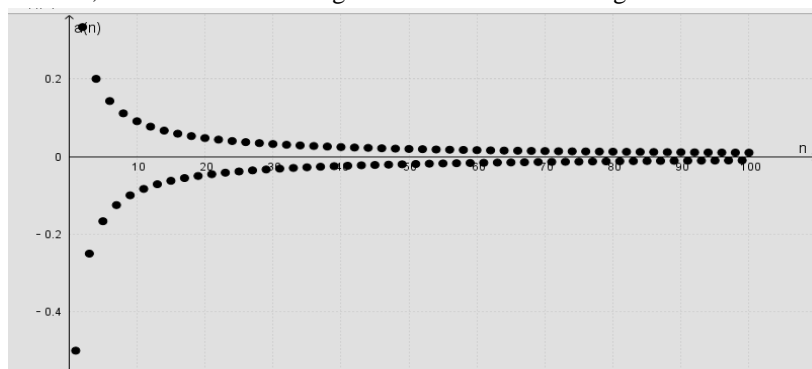


Figure 4. Change trend of series

1.3 Explain the $\varepsilon - N$ definition of sequence limit.

As can be seen from the above figure 4, as the number of items increases infinitely, there will be more and more points on the image, and eventually these points will be infinitely close to the line $x=0$. This feature of sequence is called limit, but it can not be used as a strict definition. There are two main reasons: first, by observing the change trend of the first finite term of the sequence through the image, we can infer the change trend of the later infinite term. It depends on the feeling, but the feeling is unreliable. Second, infinite increase and infinite approach are both fuzzy languages, which are not strictly defined and can not be used in logic deduction. So how to give a strict definition? Infinite approach is a qualitative description of approach state. How to use mathematical language to make quantitative analysis of approach degree?

The teacher guides the students to continue to observe the image, we can find that with the increase of the number of items, the distance between x_n and 0 is getting closer and closer, that is $|x_n - 0| \rightarrow 0$.

The quantitative analysis of $|x_n - 0| \rightarrow 0$ is carried out below.

Now choose a scale of 0.1, if the distance between 0 and x_n is less than 0.1, that is $|x_n - 0| = \left| \frac{(-1)^n}{n+1} - 0 \right| = \frac{1}{n+1} < 0.1$, we can get

$n > 9$, this shows from the tenth item, the distance between x_n and 0 is less than 0.1.

Can 0.1 describe infinite proximity? Is there a number smaller than 0.1?

The scale was further reduced to 0.01, if the distance between 0 and x_n is less than 0.01, that is $|x_n - 0| = \left| \frac{(-1)^n}{n+1} - 0 \right| = \frac{1}{n+1} < 0.01$,

we can get $n > 99$, this shows from the 100th item, the distance between x_n and 0 is less than 0.01.

Can 0.01 describe infinite proximity? Is there a number smaller than 0.01?

The scale was further reduced to 0.001, if the distance between 0 and x_n is less than 0.001, that is $|x_n - 0| = \left| \frac{(-1)^n}{n+1} - 0 \right| = \frac{1}{n+1} < 0.001$

, we can get $n > 999$, this shows from the 1000th item, the distance between x_n and 0 is less than 0.001.

No matter 0.1, 0.01 or 0.001, they can't describe the infinite proximity between x_n and 0, so the teacher introduces an arbitrarily small positive number ε . If the distance between 0 and x_n is less than the arbitrarily small positive number ε , that is

$|x_n - 0| = \left| \frac{(-1)^n}{n+1} - 0 \right| = \frac{1}{n+1} < \varepsilon$, $n > \frac{1}{\varepsilon} - 1$, $\frac{1}{\varepsilon}$ may not be an integer, in order to indicate the increasing degree of n , write down

the rounding as N , then $n > N = \left\lceil \frac{1}{\varepsilon} - 1 \right\rceil$. So we can always find an item N , and after that, it satisfies $|x_n - A| < \varepsilon$.

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If we change 0 into a constant a , and replace with a general sequence, we can get that when x_n increases infinitely, x_n is infinitely close to $a \Leftrightarrow$ when x_n increases infinitely, $|x_n - a| \rightarrow 0 \Leftrightarrow$ when x_n increases infinitely, it can be as small as you want $\Leftrightarrow |x_n - a|$ can be smaller the arbitrarily positive number ε , as long as n increases enough. Teachers can compare the process of deriving the exact definition of the limit of series to "if you want to grind a pestle into a needle, you should work hard, and you can encourage students to work hard and not afraid of difficulties." Therefore, we can get the strict definition of the limit of series: If there is a series $\{x_n\}$, if there is a constant a , there is always a arbitrarily positive number ε (no matter how small it is), there is always a positive integer N , as long as $n > N$, the corresponding x_n satisfies the inequality $|x_n - a| < \varepsilon$, the constant a is said to be the limit of the series $\{x_n\}$, or the series $\{x_n\}$ converges to a , $\lim_{n \rightarrow \infty} x_n = a$, or $x_n \rightarrow a (n \rightarrow \infty)$. If there is no such constant, series $\{x_n\}$ has no limit, or the sequence is divergent. It can be abbreviated as: $\lim_{n \rightarrow \infty} x_n = a \Leftrightarrow \forall \varepsilon > 0, \exists N \in \mathbb{Z}^+$, when $n > N$, $|x_n - a| < \varepsilon$. It can be called the $\varepsilon - N$ language of sequence limit.

In the definition, we should pay attention to:

- ① ε describes the close degree between the sequence $\{x_n\}$ and a , which has arbitrariness and certainty;
- ② N represents the increasing degree of n , which is not unique;
- ③ The logical relationship in the definition: for any given ε , find N according to $|x_n - a| < \varepsilon$, so that when $n > N$, $|x_n - a| < \varepsilon$ holds, that is, find the conditions from the conclusion, if the conditions of the conclusion can be found, then the conclusion naturally holds;
- ④ This static limit sign $\lim_{n \rightarrow \infty} x_n = a$ silently interprets the process of forever moving and infinitely approaching.

2. The Application of Limit Concept

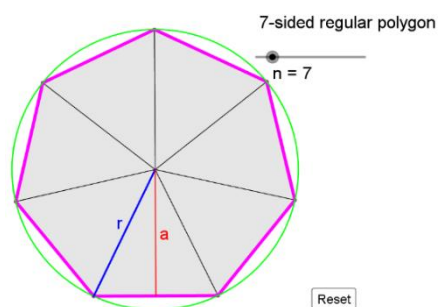


Figure 5. Area of a circle

Find the area of the circle:

$$A_{\text{polygon}} = \frac{1}{2} pa$$

When

$$n \rightarrow \infty, p \rightarrow 2\pi r$$

$$n \rightarrow \infty, a \rightarrow r$$

As shown in figure 1.4, we can get:

$$A_{\text{circle}} = \pi r^2 \quad \text{This is the area formula of a circle.}$$

RESEARCH METHODS

This research is conducted in a university in Hebei Province, hereinafter referred to as university B. The University, located in Baoding City, is a multi-disciplinary collaborative development school with teachers as the main body, science, engineering, arts, economics, management, art, education and other disciplines. The reason for choosing this school is that the author works here and is familiar with the teaching environment and teachers, which is convenient for teaching practice and communication with teachers. The research experiments mainly focus on the first semester of 2020-2021 academic year.

To understand the current situation and difficulties of students' mastery of the concept of limit, the design of this study is as follows: 80 questionnaires are distributed, the rate of recovery is 100%. The questionnaires show that students are not clear about the relationship between ε and δ and the uniqueness of N . In the learning of $\varepsilon - N$ limit, the most difficult part is to determine the value of N , and the determination of n value involves the knowledge of inequality. Therefore, the concept of inequality and its

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related knowledge will affect students' learning of strict limit language. It is difficult to separate from the details to see the overall meaning of $\varepsilon - N$ and to regard the concept of limit as a psychological object. The results also show that the connection between limit and differentiable and integral lies in limit operation, students have no impression on whether the strict limit language can describe the concept of differentiable and integral. The reason for the above phenomenon may be that daily teaching pays too much attention to operation, requires students to master various skills and methods of extreme operation, and ignores the learning of strict extreme language, thus affecting the construction of cognitive structure.

After the reform of teaching plan, by interviewing 2 teachers about the changes of students' behavior after class as well as the influence on teachers. Teachers believe that the teaching reform based on STEAM, which integrates mathematical knowledge and skills, mathematical thinking methods and mathematical development history, can cultivate students' interest in learning mathematics and better inherit mathematical culture and spirit. Also the teaching design based on STEAM education philosophy can promote the professional development of teachers.

After introducing the history of mathematics and advanced teaching technology of Geogebra, does the students' learning process improves? To answer this question, the design of this study is as follows: Based on the limit concept and APOS theory, the psychological construction process of limit concept is divided into four stages: activity stage, process stage, object stage and schema stage. The test paper is based on four stages of the set up topics for investigation, a total of 12 topics, each stage 3 topics. According to the learning requirements of limit concept in each stage, some representative test questions are selected from calculus teaching materials and related instruction books. The whole process of test questions selection, structure distribution and scoring standard is discussed and formulated by the author and many teachers with rich teaching experience, and finally determine the test papers. 39 students used the new teaching method as the experimental group, 41 students used the previous teaching method as the control group.

RESULTS

2.1 Data analysis of overall score

According to the above scoring basis, score and process the students' answers, and import the processed data into spss23.0 software to understand the overall score of the test, as shown in Figure 6, 7.

2.1.1 Histogram of total score of experimental group and control group

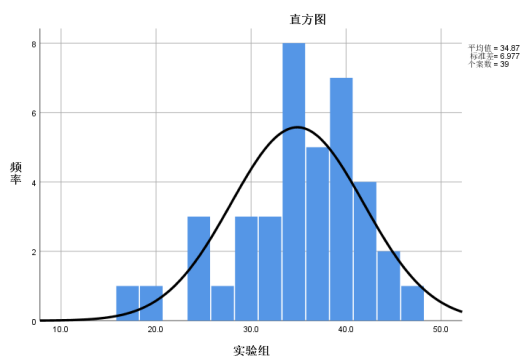


Figure 6. Histogram of experimental group

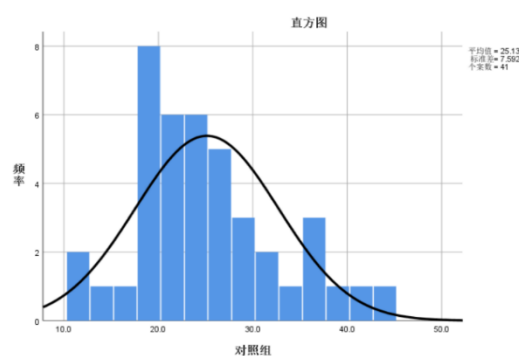


Figure 7. Histogram of control group

From the above figure 6 and table 7, they show that the students' scores in the experimental group are normal, but the tail is left. The average score of 39 students is 34.85, which is 4.85 points higher than the passing line of this test paper. The minimum score is 17 and the maximum is 46.5. The kurtosis value of 0.124 shows that the final score of the experimental group is steeper than the normal distribution, because there are more students with high scores. The skewness value of -0.688 shows that most students' scores are distributed on the right side of the average, and there are more extreme values on the left side, showing a left tail phenomenon. Combined with the numerical value, the overall score of this test is higher than the average.

The control group students appear normal but the tail to the right. The average score of 41 students is 25.13, which is less than the pass line of this test paper. The minimum score is 11.5 and the maximum is 43.5. The kurtosis value of -0.030 shows that the final score of the experimental group is close to the steepness of the normal distribution, which indicates that the distribution of performance changes on both sides of the mean value is roughly the same. The skewness value of 0.608 shows that most students' scores are distributed on the left side of the average, and there are more extreme values on the right side, showing a right tail phenomenon. Combined with the numerical value, the overall score of this test is lower than the average of 30 points.

2.1.2 Data analysis of scores in each stage

According to the test questions and scores set in the four stages, three questions are set in each stage. The full score of the activity stage is 10 points, the process stage is 10 points, and the object stage and schema stage are 15 points. The collected data of

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students' answers after quantitative and statistical processing are imported into Excel software and SPSS, the following descriptive statistics are obtained in table 2.1, 2.2.

Table 2.1. Experimental group

Stage	Case	Minimum	Maximum	Average	Standard deviation	60% of total score
Action	39	4.0	10	7.9	1.67	5.4
Process	39	0	10	7.4	2.77	6
Object	39	0	15	6.9	3.31	7.2
Scheme	39	0	15	2.9	2.89	7.2

Table 2.2. Control group

Stage	Case	Minimum	Maximum	Average	Standard deviation	60% of total score
Action	41	4.0	7	6.0	2.65	4.2
Process	41	0	6	5.3	3.00	3.9
Object	41	0	11	4.8	3.52	5.0
Scheme	41	0	12	2.7	3.02	5.

It can be seen from the above table 2.1, 2.2 that:

(1) The pass line of activity stage and process stage is 6.0 and 6.0 respectively. In the activity stage, the average score of the experimental group was 7.9, and the control group was 6.0. The standard deviation of the experimental group was 1.67, and the control group was 2.65. In the process stage, the average value of the experimental group was 7.4 points, the control group was 5.3 points, the standard deviation of the experimental group was 2.77, the control group was 3.00.

(2) The passing line of object stage and schema stage is 6.9 and 4.8 respectively. The average value of the experimental group was 6.9 points, the control group was 4.8 points, the standard deviation of the experimental group was 3.31, the control group was 3.52. In the schema stage, the average score of the experimental group was 2.9, the control group was 2.7, the standard deviation of the experimental group was 2.89, the control group was 3.02.

CONCLUSION

In this study, a series of questionnaires and tests were conducted to analyze the students' understanding of limit concept based on APOS theory. It constructs a new teaching method of calculus based on STEAM that is, introducing HPM and new teaching technology.

In the questionnaire, most students use the natural language of "tend to" or "get closer and closer" to describe the definition of sequence limit and function limit, but do not use the language of algebra or geometry. Some students think that limit is "when n tends to infinity, sequence equals a ", which is actually a static point of view. However, it is difficult for students to make dynamic analysis of limit concept in mathematical language. In this way, we can understand some typical problems of the students in ordinary colleges and universities in the concept of limit. Therefore, the design of the test paper in this study is very important.

Our construction principle of limit concept teaching is: introducing limit thought through historical development, explaining mathematical concepts with the help of historical development process of historical figures. With the help of geometric intuition, that is, the introduction of specific examples of science and technology of life to help students understand the concept of limit. Pay attention to the expression of mathematical language, strengthen students' understanding and mastery of mathematical language.

Teaching case: before teaching specific concepts, first use the examples of "circle cutting" and "stick cutting", and add the semantic explanation of limit, so that students can have a sense of the historical development of limit. At the same time, it uses the new teaching technology GeoGebra to show the students the idea of limitless approach intuitively and dynamically, and guide students to explore the relationship between "existence" and "arbitrariness". There is a progressive process in the formation of thinking. Make students have a process of exploration and understanding of $\varepsilon - N$ language, strengthen memory.

By interviewing teachers about the changes of students' behavior after class, teachers believe that the teaching reform based on STEAM, which integrates mathematical knowledge and skills, mathematical thinking methods and mathematical development history, can cultivate students' interest in learning mathematics and better inherit mathematical culture and spirit. Also the teaching design based on STEAM education philosophy can promote the professional development of teachers.

After the experimental teaching, we found that the new teaching method based on STEAM has a significant role in promoting the students' activity stage and process stage. The average value and standard deviation of the experimental group in the activity stage were 7.9 and 1.67 respectively. The average and standard deviation of the process stages are 7.4 and 2.77, respectively. The average and standard deviation of the control group in the active stage were 6.0 and 2.65 respectively. The average and standard deviation of the process stages are 5.3 and 3.00 respectively.

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The new teaching method based on STEAM also slightly promotes the students' object stage and schema stage, but the effect is not very obvious. The average and standard deviation of the experimental group in the object stage were 6.9 and 3.31, respectively. The average value and standard deviation of schema stage are 2.9 and 2.89 respectively. The average and standard deviation of the control group were 4.8 and 3.52 respectively. The average value and standard deviation of schema stage are 2.7 and 3.02 respectively.

The introduction of HPM and GeoGebra has a positive impact on the overall performance of students. The experimental group showed the phenomenon of normal right in the test results, that is, the actual score of students was higher than the average score, which performs better than the control group.

STEAM education concept is integrated into mathematics teaching reform, STEAM content is integrated around limit concept, and learning effect is evaluated by stages based on APOS theory. Practice has proved that STEAM education concept is very important for the cultivation of practical and innovative talents. The vivid cases of extreme teaching show how to integrate mathematics education into STEAM education. In the teaching of limit concept, we should integrate the resources of mathematics culture and information technology.

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